BELLCOMM, INC.
1100 Seventeenth Street, N.W. Washington, D.C. 20036

SUBJECT: A Study of Methods of Augmenting Cross-Product Steering with Direct Control of Out-of-Plane Position Errors - CSM Transearth Injection
Case 310

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FROM: D. A. Corey

TM-67-2012-3

TECHNICAL MEMORANDUM

INTRODUCTION

This memorandum is the third in a series which discuss the relative merits of augmenting cross-product steering with direct control of out-of-plane position errors (yawsteering). The first memorandum (Reference 1) of the series discussed the methods used in the studies. The second memorandum (Reference 2) discussed the results of the study for the Lunar Orbit Insertion maneuver. This memorandum will discuss the results obtained when the various yawsteering methods were used in the Transearth Injection (TEI) maneuver.

The study consisted of comparing the results obtained by cross-product steering alone with the results obtained using various methods of yawsteering. The results of interest will be the fuel required by each steering method during the maneuver, the relative accuracy of the steering method, and the midcourse corrections required.

As a by product of the study, some data was produced which compares the effect of two different aim points for the TEI maneuver and a comparison of two values of the cross-product steering law constant c.

REFERENCE TRAJECTORIES

Three different trajectories were used as the reference trajectories in this study. They are described in Reference 3. Table 1 presents a brief summary of the parameters of these trajectories for the TEI phase. The three trajectories were selected to test the yawsteering methods over a range of plane change requirements. Generally, the Apollo TEI maneuver does not require a large plane change so the range tested is smaller than was tested for the Lunar Orbit Insertion maneuver. The largest, trajectory 4.1, requires a plane change of 4.18°, trajectories 4.4 and 4.5 require plane changes of .35° and 1.35° respectively.

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METHODS EMPLOYED

The powered flight transition matrices for each of the cases studied were developed using the Bellcomm Powered Flight Performance Simulator program (Reference 4). A description of the pertinent details of this program is contained in Reference 1. The transition matrices relate deviations in the vehicle state vector 10 seconds after nominal engine cutoff to deviations in the vehicle state vector at the beginning of the TEI maneuver and to deviations in the vehicle performance and the inertial measurement system. The transition matrices were used to propagate covariance matrices of errors which existed at engine ignition through the burn developing the covariance matrices of the actual deviations and the uncertainties in the deviations after the burn.

An excellent measure of the comparative accuracy of the guidance schemes studied is the transearth midcourse corrections required for each of the guidance schemes. Accordingly, the covariance matrices representing the actual errors at the end of the TEI maneuver were propagated out to the Moon's sphere of action and the midcourse correction requirements at that point were determined. The criteria for the correction was to achieve the reference trajectory Earth Entry position (400,000 feet altitude) at the reference time. The covariance matrices were then modified by the correction covariance matrix and propagated on to the Earth Entry point. A second "midcourse" correction was applied at that point. The second correction simply nulls the velocity errors at the Entry point. The second correction is not, of course, a powered maneuver at all, but it provides the covariance matrix of velocity errors at the Entry point. This method provides the midcourse requirements due to errors in the state vector before TEI and due to vehicle performance and sensing errors during TEI.

Another method of determining the midcourse correction statistics was also employed. In this method the actual state vector deviations at the end of TEI due to each of the vehicle performance and sensing errors was propagated out to the Moon's sphere of action and, after the correction was applied, on to Earth Entry. This method allowed the separate evaluation of the contribution of each individual error source. The method, however, assumes statistical independence of the individual error sources and cannot be used to propagate state vector errors existing prior to TEI ignition.

In both methods, perfect midcourse navigation and perfect correction execution were assumed.

The covariance matrices representing the errors existing prior to TEI were developed in the study reported in Reference 5. The case selected for use in this study assumed that Lunar Orbit Navigation was performed using the on-board optical navigation system with a sextant having an accuracy three times worse than the design value. It was further assumed that a state vector update and a corresponding retargeting were performed immediately prior to TEI ignition. This resulted in the covariance matrix of actual errors being equal to the covariance matrix of uncertainties in the state vector, and the two matrices have perfect negative correlation between them.

The transearth free flight propagation was based on linear transition matrices developed from integrated perturbations about the reference trajectory.

In both methods above, covariance matrices of the required midcourse corrections were formed and then used to determine the statistical distribution of the magnitude of the required midcourses. The 99.73% points of these distributions were then determined and these values are reported in the tables accompanying this memorandum.

ERROR SOURCES CONSIDERED

Table 2 presents a list of the error sources considered in the simulations. The magnitude of the platform errors are the same as those used in Reference 6 except that the gyro bias drift errors used were 5 times greater than those used in Reference 6. One sigma values of thrust, initial mass, and engine specific impulse errors are all 1% of the nominal values. For purposes of developing the powered flight transition matrices, the assumed initial position deviations and uncertainties were 10,000 feet in the radial, downrange, and out-of-plane directions, and the initial velocity deviations and uncertainties were 10 feet per second in the radial, downrange and out-of-plane directions.

The time of engine ignition uncertainty simulates the uncertainty in the engine thrust buildup characteristics. A value of .01 seconds was used as the one sigma value. The time of engine cutoff uncertainty includes the effects of the uncertainty in the engine thrust tailoff characteristics, the fact that the engine off signal can only be sent at certain discrete times, and any scheme error involved in computing the desired engine off time. The one sigma value used for this error source was .01414 seconds.

GUIDANCE EQUATIONS

The cross-product steering law and the yawsteering equations used are discussed in Reference 1. The required velocity vector, $\underline{V}_r$ is defined for TEI as that velocity which, if achieved instantaneously, would place the vehicle on a hyperbola which passes through a desired aim point at a specified time. In effect, $\underline{V}_r$ is the solution to Lambert's problem - given two points and a time of flight between them, determine the conic (and therefore the necessary current velocity) between the points. The equations and the iteration scheme used are essentially the same as those described in Reference 7.

Two aim points were selected for use in the study. Both points lie on the osculating hyperbola passing through the nominal TEI engine cutoff position. One case considered a point at a time equivalent to the time at which the first mid-course correction was to be made, i.e., the Moon's sphere of action. This time was 8 hours 20 minutes for trajectory 4.1 and 10 hours for trajectories 4.4 and 4.5. The other aim point corresponded to the point on the hyperbola at the time of Earth Entry. These points will be referred to as the MSA aim point and the Entry aim point respectively.

Two values of the guidance law constant, c, (see Reference 1) were selected for study. They were 1.0 and 0.5.

STABLE MEMBER AXIS ALIGNMENT

The stable member axis alignment defined for TEI and used in this study is as follows:

$$\underline{X}_{SM} = \underline{i}_T$$

$$\underline{Y}_{SM} = \underline{Z}_{SM} \times \underline{X}_{SM}$$

$$\underline{Z}_{SM} = \text{UNIT}\ (\underline{i}_T \times \underline{r})$$

where $\underline{i}_T$ is the unit thrust acceleration vector at engine ignition
and $\underline{r}$ is the vehicle position vector at engine ignition.

DESCRIPTION OF DATA TABLES

Table 3 contains a list of the cases studied in terms of the reference trajectory identification, type of steering used, aim point, guidance constant, and type of time to go calculation used (see Reference 1). The cases are numbered to facilitate and abbreviate the discussion which follows.

Tables 4 through 11 present the numerical results of the study. Table 4 presents the nominal (no errors) TEI final conditions for each of the cases studied in terms of several parameters of interest.

Table 5 presents the standard deviations of actual orbital element errors at the end of TEI. These values include the effects of all error sources except the position and velocity deviations and uncertainties at TEI ignition.

Table 6 presents the standard deviations of the errors in the actual (real world) state vectors at the end of the TEI maneuvers. Table 7 presents the standard deviations of the errors in the estimated (sensed by the guidance system) state vectors at the end of TEI. Table 8 presents the standard deviations of the uncertainties (difference between the actuals and the estimates) in the state vector errors. These 3 tables present the data in terms of the 3 components of position and of velocity and the fuel mass required. The coordinate system used is the UVW, or orbit plane coordinate system defined with U - the radial component, V - the downrange component and W - the out-of-plane component (see Reference 1 for details).

Table 9 describes the statistics of the first midcourse correction due to inertial platform and vehicle performance errors only. A limited number of the errors were found to be the dominant contributors and the 1σ values of the corrections due to these errors are listed. In addition, the 99.73% (equivalent 3σ) point of the correction magnitude distribution is presented.

Table 10 presents the same data as Table 9 but for the second midcourse correction.

Table 11 presents the 99.73% points of the midcourse correction magnitudes including the effects of the state vector errors existing prior to TEI ignition. Actually, considering these state vector errors had very little effect on the midcourse correction requirements even for the rather pessimistic case assumed. The figures presented in Table 11 are only very slightly larger than the comparable figures in Tables 9 and 10, indicating that the errors which occurred during the burn itself completely dominated the final errors. Consequently, little more will be said of Table 11 though it is included for the reader's information.

DISCUSSION OF RESULTS

Discussion of the various aspects of the results follows.

No Yawsteering Vs. Linear Yawsteering

(Cases 1 vs. 2, 4 vs. 5, 8 vs. 9, 11 vs. 12, 13 vs. 14, 15 vs. 16)

There was basically no difference in the nominal results for the two steering methods. Burn times differed only by the order of a few thousandths of a second and consequently there was no nominal fuel penalty for linear yawsteering and no difference in the achieved orbit.

The linear yawsteering law did provide a slight reduction in the orbit inclination error due to all error sources. The improvement was of the order of 5% for trajectory 4.1, 1.4% for trajectory 4.4 and 0% for trajectory 4.5. Similarly, the error in the longitude of the ascending node error is reduced somewhat by the linear yawsteering method.

The in-plane components of the actual and estimated errors differed very slightly between the two methods. Most changes were of the order of 1% or so and may be considered negligible.

The linear yawsteering law did provide a substantial percentage reduction in the actual and estimated out-of-plane position errors in the trajectory 4.1 and 4.5 cases, though the errors were fairly small for the no yawsteering cases so that the improvement was of small value. The actual out-of-plane error was slightly larger for trajectory 4.4 with yawsteering though the estimated out-of-plane error was reduced by yawsteering. This effect was produced by two out-of-plane accelerometer error sources - z accelerometer misalignment in the XZ plane, and z accelerometer bias. The yawsteering law senses an erroneous output from the z accelerometer and attempts to correct for the error resulting in an increase in the out-of-plane position error. This effect is noticeable only on trajectories involving very small plane changes because it is swamped out by out-of-plane errors due to thrust and mass deviations on trajectories involving a larger plane change. The actual error produced is very small however, and the only importance of the effect is as a pathological curiosity.

The uncertainties in the end conditions were unaffected by the steering method.

Linear yawsteering resulted in very slightly smaller midcourse corrections. The largest decrease was about 1% for case 4 vs. case 5. The improvement can be considered negligible.

Linear Yawsteering Vs. Quadratic Yawsteering

Case 2 vs. 3, and 9 vs. 10)

The quadratic yawsteering law required a slightly longer (0.2%) burn time for the nominal case. The in-plane actual and estimated errors were changed very slightly and in fact had almost the same values as the no yawsteering cases. The out-of-plane errors were virtually identical for the two yawsteering laws as were the uncertainties in the errors. The magnitudes of the midcourse corrections were very slightly larger (0.3%) for the first correction and 0.6% for the second). This was primarily due to an increase in the ΔV required to make up for the cutoff time uncertainty (TIMCOU). This was due to the fact that the quadratic yawsteering law produces zero out-of-plane acceleration at the end of the burn. Consequently, all of the extra engine ΔV produced by the cutoff error is in plane where it costs the most in midcourse ΔV.

Entry Aim Point Vs. MSA Aim Point

(Cases 1 vs. 8, 2 vs. 9, 3 vs. 10, 4 vs. 11, 5 vs. 12)

The MSA aim point yielded a very slightly lower (50 feet) nominal perilune altitude at engine cutoff and in the cases where yawsteering was not employed, the errors in orbital inclination were smaller by 1%. The MSA aim point constrains the inclination more than the Entry aim point because the angle between TEI and MSA is closer to 90°. This argument can be most easily seen by considering an aim point 180° away. Such an aim point can be achieved with any inclination. Accordingly, for the cases without yawsteering, the MSA aim point yielded smaller out-of-plane position and velocity errors. No difference in these errors was present in the yawsteering cases because plane control was achieved independent of the aim point.

Some variation in the inplane components of the errors was noted. The most significant change was the U or radial position component error which was 1 to 3% higher for the MSA aim point. The thrust and mass deviation errors caused most of the difference.

The uncertainties were very nearly identical for both aim points.

The magnitudes of the midcourse corrections were not significantly different for either of the two aim points.

Guidance Constant c = 1.0 vs. c = 0.5

(Cases 1 vs. 4, 2 vs. 5, 8 vs. 11, 9 vs. 12)

The c = 0.5 case resulted in a significantly lower nominal perilune altitude. The difference amounted to about 1550 feet. It is, however, well known that the value of the guidance constant affects the pericenter altitude.

The actual altitude (U) errors were larger by 15% for the c = 0.5 cases. This was a direct effect of the guidance law. The greater altitude errors result from those error sources which affect the sensed vehicle acceleration such as engine thrust and vehicle mass deviations, x (thrust axis) accelerometer scale factor and linearity. The increase in the U component was generally accompanied by a negatively correlated increase in the $\dot{V}$ or downrange velocity component for the error sources. There is no net effect on the "quality" of the achieved orbit in terms of how well the aim point is achieved. This is evidenced by the fact that the midcourse corrections for these error sources were not appreciably affected by the increased deviations at the end of TEI.

In general, there was no significant difference in the midcourse requirements for the two values of the guidance constant.

Copp's Time-to-go Computation Vs. $T_{go} = Vg/\dot{V}g$

(Cases 2 vs. 6, 5 vs. 7)

The relative merits of Copp's rather elaborate, but accurate time-to-go calculation (see Reference 1) versus a simpler though less accurate calculation based on the velocity to be gained and its time derivative were evaluated for two of the linear yawsteering cases.

The simpler time-to-go calculation required a very slightly longer burn time. The difference in the ΔV for the maneuver amounted to about 1.5 feet per second out of the total of 2900 feet per second required for the maneuver. Other parameters of the nominal achieved orbit were not affected however.

The error statistics at the end of TEI were very nearly identical for both methods and, consequently, so were the midcourse statistics. The simpler time-to-go calculation involves no significant penalty.

DOMINANT ERROR SOURCES

In the course of the study, a small number of the error sources were found to be dominant in that they were responsible for considerably larger midcourse corrections than the rest of the error sources. Tables 9 and 10 present the one sigma values of the error sources which required greater than .25 feet per second, one sigma, for either of the two corrections.

The error source causing the largest corrections was the X-accelerometer bias. This accelerometer lies along the thrust axis of the vehicle at engine ignition and consequently senses most of the thrust acceleration during the maneuver. The error source produces a considerable uncertainty in $\dot{V}$, the downrange velocity error at the end of TEI. The result is a one sigma midcourse penalty of about 1.85 fps in the trajectory 4.1 cases and slightly greater for the other two cases where the first midcourse was made a little later in time. The X-accelerometer scale factor error was substantial for the same reasons and the one sigma first midcourse penalties amounted to about 0.9 fps.

The second largest penalties were caused by the z gyro (pitch axis) constant drift term. It was assumed that the platform was aligned 15 minutes prior to engine ignition. The penalties would be proportionately larger for longer drift times. The one sigma first midcourse value was about 1.73 fps for trajectory 4.1 and slightly less for the other two trajectories. The Y gyro (yaw axis) constant drift term was almost as large in the case of trajectory 4.1. The first midcourse penalty was about .94 fps. The effect of the yaw gyro was considerably smaller for the two trajectories involving much smaller plane changes however. The one sigma first midcourse penalties were .36 fps and .46 fps for trajectories 4.4 and 4.5 respectively.

The time of engine cutoff error was also a strongly dominant error source causing a first midcourse penalty of about 1.25 fps. A documented estimate of the value of this error source was not available but the importance of the error source is demonstrated for the value of .01414 seconds assumed.

The effect of initial platform misalignment about the pitch (Z) axis was a significant contributor to the midcourse penalties. This error source caused a one sigma first midcourse penalty of about 1 fps. Initial platform misalignment about the yaw (Y) axis caused a somewhat smaller first midcourse penalty, about .57 fps for the trajectory 4.1 cases and about .25 fps for the two small plane change trajectories.

The magnitudes of the second midcourse corrections generally followed the same pattern as the first midcourses. The choice of placing the second correction at the Entry point provides information of the velocity errors at that point but the figures are not useful for determining midcourse ΔV budgets. Almost all of the second midcourse ΔV was in the out-of-plane direction, i.e., azimuth errors were considerably greater than flight path angle errors. There were, of course, no position errors since perfect first correction execution was assumed.

CONCLUSIONS

The study has demonstrated that augmenting cross-product steering with direct control of the out-of-plane errors offers little or no advantage for the plane changes generally required in the Transearth Injection maneuver. This, together with the fact that the yawsteering logic would require additional on-board computer storage certainly argues against its use for the Apollo TEI maneuver.

It was also found that using a guidance aim point on the desired hyperbola corresponding to the time of Earth Entry produced results virtually identical to those obtained using an aim point corresponding to the time of Moon's sphere of action passage.

The midcourse statistics for a cross-product guidance law constant of 0.5 were not significantly different from these for a guidance constant of 1.0 although there was a difference in the perilune altitude of the achieved orbit and c = 0.5 case produced somewhat larger altitude errors at injection.

The three sigma magnitudes of the required midcourse corrections (when made at the Moon's sphere of action and targeted to achieve the reference Entry point at the reference time) were found to be 9.2 fps for trajectory 4.1, 9.7 fps for trajectory 4.4 and 9.0 fps for trajectory 4.5. The velocity errors at the Entry point based on the single correction were 13.0 fps, 8.6 fps and 9.1 fps for trajectories 4.1, 4.4, and 4.5 respectively. The Entry velocity errors were almost entirely in the out-of-plane direction. These figures are somewhat optimistic since they include neither the effects of errors in the execution of the midcourse correction nor navigation errors in the Transearth phase.

D. A. Corey

2012-DAC-vh D. A. Corey

Attachments
Tables 1 thru 11

REFERENCES

1. A Study of Methods of Augmenting Cross Product Steering with Direct Control of Out-of-Plane Position Errors - Description of the Methods Used; TM-66-2012-6, D. A. Corey, Bellcomm, Inc., November 25, 1966.

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3. Introduction to the Issue 4 Reference Trajectory Data Package, TR-66-209-6, R. L. Wagner, Bellcomm, Inc., September 22, 1964.

4. Bellcomm Powered Flight Performance Simulator, D. A. Corey, B. G. Niedfeldt, D. J. Roek, Bellcomm, Inc., January 12, 1967.

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6. Master End Item Detail Specification, (Prime Equipment Performance/Design and Product Configuration Requirements), Part I, MEI No. 2015000, Airborn-Guidance and Navigation Equipment Block II for Apollo Command Module, MIT/IL June 21, 1965.

7. Guidance System Operations Plan AS-278 Vol. 1, CM GNCS Operations, R-547, MIT/IL, October 1966.

TABLE 1

Summary of Reference Trajectory and Vehicle Parameters

	Traj. 4.1	Traj. 4.4	Traj. 4.5
Lunar Parking Orbit			
Selenographic Inclination (degrees)	174.98	173.15	175.25
Selenographic Longitude of the Ascending Node (degrees)	209.07	156.68	19.25
Altitude (feet)	485486	483630	484902
Osculating Hyperbola at Injection			
Selenographic Inclination (degrees)	178.88	173.01	173.98
Selenographic Longitude of the Ascending Node (degrees)	247.96	159.34	24.36
Eccentricity	1.3774	1.2416	1.2393
Altitude of Perilune (feet)	485128	482832	484936
Selenographic Injection Coordinates			
Latitude (degrees)	.611	-6.414	2.069
Longitude (degrees)	-157.904	-133.870	-134.983
Injection Altitude (feet)	496624	486734	491974
Plane Change in TEI (degrees)	4.18	0.35	1.35
Free Flight Time to Entry (hours)	75.96	109.18	109.70
Nominal Engine Thrust (pounds)	21900	21900	21900
Nominal Engine Specific Impulse (sec.)	313	313	313
Vehicle Mass at Ignition (slugs)	896.804	1080.84	920.492

TABLE 2

ERROR SOURCES

SYMBOL	DESCRIPTION
GMUX	X-Gyro Mass Unbalance
GMUY	Y-Gyro Mass Unbalance
GMUZ	Z-Gyro Mass Unbalance
INMISX	Platform Misalignment About the X-Gyro Input Axis
INMISY	Platform Misalignment About the Y-Gyro Input Axis
INMISZ	Platform Misalignment About the Y-Gyro Input Axis
ASFX	X-Accelerometer Scale Factor
ASFY	Y-Accelerometer Scale Factor
ASFZ	Z-Accelerometer Scale Factor
ACLINX	X-Accelerometer Linearity
ACLINY	Y-Accelerometer Linearity
ACLINZ	Z-Accelerometer Linearity
AMSXXY	X-Accelerometer Misalignment in the X-Y Plane
AMSXXZ	Y-Accelerometer Misalignment in the X-Z Plane
AMSYXY	Y-Accelerometer Misalignment in the X-Y Plane
AMSYYZ	Y-Accelerometer Misalignment in the Y-Z Plane
AMSZXZ	Z-Accelerometer Misalignment in the X-Z Plane
AMSZYZ	Z-Accelerometer Misalignment in the Y-Z Plane

TABLE 2

(continued)

SYMBOL	DESCRIPTION
GDSAX	X-Gyro Drift About the Spin Axis
GDSAY	Y-Gyro Drift About the Spin Axis
GDSAZ	Z-Gyro Drift About the Spin Axis
GCDX	X-Gyro Constant Drift About the Input Axis*
GCDY	Y-Gyro Constant Drift About the Input Axis*
GCDZ	Z-Gyro Constant Drift About the Input Axis*
ACBIAX	X-Accelerometer Bias
ACBIAY	Y-Accelerometer Bias
ACBIAZ	Z-Accelerometer Bias
TIMIGU	Ignition Time Uncertainty
TIMCOU	Engine Cutoff Time Uncertainty
FTD	Thrust Magnitude Deviation
BID	Engine I_{sp} Deviation
MASSD	Vehicle Initial Mass Deviation
PIUU	Initial Position Uncertainty in the Vertical Direction
PIUV	Initial Position Uncertainty in the Downrange Direction
PIUW	Initial Position Uncertainty in the Out-of-Plane Direction

*The platform is assumed to be aligned 45 minutes prior to engine ignition

TABLE 2

(continued)

SYMBOL	DESCRIPTION
VIUU	Initial Velocity Uncertainty in the Vertical Direction
VIUV	Initial Velocity Uncertainty in the Downrange Direction
VIUZ	Initial Velocity Uncertainty in the Out-of-Plane Direction
PIDU	Initial Position Deviation in the Vertical Direction
PIDV	Initial Position Deviation in the Downrange Direction
PIDW	Initial Position Deviation in the Out-of-Plane Direction
VIDU	Initial Velocity Deviation in the Vertical Direction
VIDV	Initial Velocity Deviation in the Downrange Direction
VIDW	Initial Velocity Deviation in the Out-of-Plane Direction

TABLE 3

Summary of the Cases Studied

Case No.	Trajectory No.	Steering Law	Aimpoint	Guidance Constant	Time-to-Go Computation
1	4.1	No Yawsteering	Entry	1.0	Copps
2	4.1	Linear Yaw-steering	Entry	1.0	Copps
3	4.1	Quadratic Yaw-steering	Entry	1.0	Copps
4	4.1	No Yawsteering	Entry	0.5	Copps
5	4.1	Linear Yaw-steering	Entry	0.5	Copps
6	4.1	Linear Yaw-steering	Entry	1.0	$VG/\dot{V}G$
7	4.1	Linear Yaw-steering	Entry	0.5	$VG/\dot{V}G$
8	4.1	No Yawsteering	MSA	1.0	Copps
9	4.1	Linear Yaw-steering	MSA	1.0	Copps
10	4.1	Quadratic Yaw-steering	MSA	1.0	Copps
11	4.1	No Yawsteering	MSA	0.5	Copps
12	4.1	Linear Yaw-steering	MSA	0.5	Copps
13	4.4	No Yawsteering	Entry	1.0	Copps
14	4.4	Linear Yaw-steering	Entry	1.0	Copps
15	4.5	No Yawsteering	Entry	1.0	Copps
16	4.5	Linear Yaw-steering	Entry	1.0	Copps

TABLE 4

Nominal TEI Final Conditions

Case No	Case Description	Burn Time (sec)	ΔV (fps)	Eccentricity	Inclination (degrees)	Longitude of Ascending Node (degrees)	Perilune Altitude (feet)
1	4.1 No Yawsteering C=1 Copps Entry	103.417	2907.4	1.3775	178.88	247.96	487920
2	4.1 Linear Yawsteering C=1 Copps Entry	103.419	2907.4	1.3775	178.88	247.96	487920
3	4.1 Quadratic C=1 Copps Entry	103.660	2915.4	1.3795	178.88	247.96	487910
4	4.1 No Y.S. C=0.5 Copps Entry	103.429	2907.8	1.3774	178.88	247.98	486330
5	4.1 Linear Y.S. C=0.5 Copps Entry	103.429	2907.8	1.3774	178.88	247.96	486340
6	4.1 Linear Y.S. C=1. VG/$\dot{V}$G Entry	103.464	2909.0	1.3775	178.88	247.96	487910
7	4.1 Linear Y.S. C=0.5 VG/$\dot{V}$G Entry	103.461	2908.9	1.3774	178.88	247.96	486340
8	4.1 No Y.S. C=1.0 Copps MSA	103.415	2907.4	1.3776	178.88	247.95	487870
9	4.1 Linear Y.S. C=1.0 Copps MSA	103.418	2907.5	1.3776	178.88	247.96	487870
10	4.1 Quadratic Y.S. C=1. Copps MSA	103.670	2915.7	1.3776	178.88	247.96	487860
11	4.1 No Y.S. C=.5 Copps MSA	103.427	2907.7	1.3774	178.88	247.94	486330
12	4.1 Linear Y.S. C=.5 Copps MSA	103.426	2907.7	1.3774	178.88	247.96	486330
13	4.4 No Y.S. C=1.0 Copps Entry	114.294	2631.5	1.2417	173.01	159.34	485640
14	4.4 Linear Y.S. C=1.0 Copps Entry	114.297	2631.5	1.2417	173.01	159.34	485640
15	4.5 No Y.S. C=1.0 Copps Entry	97.305	2630.4	1.2393	173.99	24.356	487330
16	4.5 Linear Y.S. C=1.0 Copps Entry	97.308	2630.4	1.2393	173.98	24.356	487330

TABLE 5

Standard Deviations of Actual Orbital Element Errors at the End of TEI

CASE NO.	CASE DESCRIPTION	ECCENTRICITY	INCLINATION (Degrees)	LONGITUDE OF THE ASCENDING NODE (Degrees)	PERICENTER ALTITUDE (Feet)
1	4.1 No Yawsteering C=1 Copps Entry	5.398×10^{-4}	.0120	.6612	38.986
2	4.1 Linear Yaw-steering C=1 Copps Entry	5.378×10^{-4}	.0113	.6625	38.191
3	4.1 Quadratic C=1 Copps Entry	5.406×10^{-4}	.0113	.6619	38.293
4	4.1 Nc Yawsteering C=.5 Copps Entry	5.476×10^{-4}	.0119	.6659	46.308
5	4.1 Linear Yaw-steering C=.5 Copps Entry	5.304×10^{-4}	.0114	.6615	46.606
6	4.1 Linear Yaw-steering C=1. VG/VG Entry	5.337×10^{-4}	.0114	.6614	37.823
7	4.1 Linear Yaw-steering C=.5 VG/VG Entry	5.476×10^{-4}	.0114	.6616	48.052
8	4.1 No Yawsteering C=1.0 Copps MSA	5.403×10^{-4}	.0118	.6626	38.187

TABLE 5 (Contd.)

Standard Deviations of Actual Orbital Element Errors at the End of TEI

CASE NO.	CASE DESCRIPTION	ECCENTRICITY	INCLINATION (Degrees	LONGITUDE OF THE ASCENDING NODE (Degrees)	PERICENTER ALTITUDE (Feet)
9	4.1 Linear Yaw-steering C=1.0 Copps MSA	5.390×10^{-4}	.0113	.6630	38.429
10	4.1 Quadratic Y.S. C=1. Copps MSA	5.416×10^{-4}	.0113	.6620	38.401
11	4.1 No Yawsteering C=.5 Copps MSA	5.392×10^{-4}	.0118	.6656	46.562
12	4.1 No Yawsteering C=.5 Copps MSA	5.389×10^{-4}	.0113	.6623	47.202
13	4.4 No Yawsteering C=1.0 Copps Entry	5.030×10^{-4}	.0073	.1208	108.34
14	4.4 Linear Yaw-steering C=1.0 Copps Entry	4.964×10^{-4}	.0072	.1217	108.90
15	4.5 No Yawsteering C=1.0 Copps Entry	4.691×10^{-4}	.0153	.0487	34.794
16	4.5 Linear Yaw-steering C=1.0 Copps Entry	4.716×10^{-4}	.0153	.0482	34.016

TABLE 6

Standard Deviations of the Actual Errors at the End of TEI

CASE NO.	CASE DESCRIPTION	U (feet)	V (feet)	W (feet)	$\dot{U}$ (fps)	$\dot{V}$ (fps)	$\dot{W}$ (fps)	MASS (slugs)
1	4.1 No Yawsteering C=1 Copps Entry	171.4	2127.1	353.7	2.730	.951	2.467	8.022
2	4.1 Linear Yawsteering C-1 Copps Entry	173.5	2166.7	143.8	2.737	.948	2.444	7.019
3	4.1 Quadratic C-1 Copps Entry	172.8	2131.2	144.0	2.732	.952	2.444	6.992
4	4.1 No Yawsteering C=.5 Copps Entry	198.3	2124.3	340.5	2.731	.966	2.465	7.012
5	4.1 Linear Yawsteering C=.5 Copps Entry	200.6	2162.2	143.8	2.757	.935	2.445	7.041
6	4.1 Linear Yawsteering C=1. $V\dot{G}/VG$ Entry	174.1	2170.3	143.9	2.744	.941	2.446	7.098
7	4.1 Linear Yawsteering C=.5 $V\dot{G}/VG$ Entry	201.0	2165.7	143.9	2.750	.967	2.446	7.041
8	4.1 No Yawsteering C=1.0 Copps MSA	173.4	2127.4	347.2	2.717	.952	2.458	7.017
9	4.1 Linear Yawsteering C=1.0 Copps MSA	175.4	2166.2	143.8	2.728	.950	2.444	7.020
10	4.1 Quadratic Yawsteering C=1. Copps MSA	174.9	2130.6	144.0	2.722	.955	2.444	7.042

TABLE 6 (Cont'd)

Standard Deviations of the Actual Errors at the End of TEI

CASE NO.	CASE DESCRIPTION	U (feet)	V (feet)	W (feet)	$\dot{U}$ (fps)	$\dot{V}$ (fps)	$\dot{W}$ (fps)	MASS (slugs)
11	4.1 No Yawsteering C=.5 Copps MSA	200.0	2124.4	336.5	2.724	.951	2.457	7.015
12	4.1 Linear Yawsteering C=.5 Copps MSA	201.9	2161.7	143.8	2.733	.951	2.445	7.019
13	4.4 No Yawsteering C=1.0 Copps Entry	143.4	2150.6	142.8	2.591	.892	2.264	8.625
14	4.4 Linear Yawsteering C=1.0 Copps Entry	142.8	2150.8	146.3	2.590	.879	2.277	8.642
15	4.5 No Yawsteering C=1.0 Copps Entry	130.9	1816.2	156.8	2.447	.844	2.225	7.355
16	4.5 Linear Yawsteering C=1.0 Copps Entry	131.2	1820.5	125.2	2.449	.849	2.228	7.340

TABLE 7

Standard Deviations of the Estimated Errors at the End of TEI

CASE NO.	CASE DESCRIPTION	U (feet)	V (feet)	W (feet)	$\dot{U}$ (fps)	$\dot{V}$ (fps)	$\dot{W}$ (fps)	MASS (Slugs)
1	4.1 No Yawsteering C=1 Copps Entry	101.6	2127.3	326.1	1.171	.456	.392	7.022
2	4.1 Linear Yawsteering C=1 Copps Entry	104.9	2166.8	0.7	1.190	.455	.068	7.019
3	4.1 Quadratic C=1 Copps Entry	103.3	2131.3	0.3	1.179	.461	.011	6.992
4	4.1 No Yawsteering C=.5 Copps Entry	142.2	2124.5	311.7	1.178	.456	.374	7.012
5	4.1 Linear Yawsteering C=.5 Copps Entry	145.1	2162.4	0.7	1.231	.455	.068	7.041
6	4.1 Linear Yawsteering C=1. VG/$\dot{V}$G Entry	105.3	2170.5	1.0	1.203	.450	.097	7.098
7	4.1 Linear Yawsteering C=.5 VG/$\dot{V}$G Entry	145.6	2165.9	1.0	1.213	.452	.097	7.041
8	4.1 No Yawsteering C=1.0 Copps MSA	104.4	2127.6	319.0	1.143	.456	.317	7.017
9	4.1 Linear Yawsteering C=1.0 Copps MSA	107.7	2166.4	0.7	1.164	.455	.068	7.020
10	4.1 Quadratic Yawsteering C=1. Copps MSA	106.3	2130.8	0.3	1.146	.461	.011	7.042

TABLE 7 (Cont'd)

Standard Deviations of the Estimated Errors at the End of TEI

CASE NO.	CASE DESCRIPTION	U (feet)	V (feet)	W (feet)	$\dot{U}$ (fps)	$\dot{V}$ (fps)	$\ddot{W}$ (fps)	MASS (slugs)
11	4.1 No Yawsteering C=.5 Copps MSA	144.3	2124.6	307.3	1.157	.456	.305	7.015
12	4.1 Linear Yawsteering C=.5 Copps MSA	147.2	2161.9	0.7	1.178	.456	.068	7.019
13	4.4 No Yawsteering C=1.0 Copps Entry	37.4	2151.0	40.86	1.214	.379	.062	8.625
14	4.4 Linear Yawsteering C=1.0 Copps Entry	36.6	2151.2	.2	1.187	.377	.005	8.642
15	4.5 No Yawsteering C=1.0 Copps Entry	53.3	1816.4	101.9	1.000	.436	.154	7.355
16	4.5 Linear Yawsteering C=1.0 Copps Entry	53.8	1820.8	.3	1.010	.437	.023	7.340

TABLE 8

Standard Deviations of the Uncertainty of the Errors at the End of TEI

Case No.	Case Description	U (feet)	V (feet)	W (feet)	$\dot{U}$ (fps)	$\dot{V}$ (fps)	$\dot{W}$ (fps)
1	4.1 No Yawsteering C=1 Copps Entry	144.1	50.9	143.8	2.464	.836	2.443
2	4.1 Linear Yaw-steering C=1 Copps Entry	144.2	50.9	143.8	2.464	.835	2.443
3	4.1 Quadratic C=1 Copps Entry	144.5	51.1	144.0	2.465	.836	2.444
4	4.1 No Yawsteering C=.5 Copps Entry	144.4	50.8	143.8	2.465	.836	2.444
5	4.1 Linear Yaw-steering C=.5 Copps Entry	144.7	50.6	143.9	2.465	.836	2.444
6	4.1 Linear Yaw-steering C=1 $VG/\dot{V}G$ Entry	144.7	50.9	143.9	2.465	.835	2.444
7	4.1 Linear Yaw-steering C=.5 $VG/\dot{V}G$ Entry	145.0	50.6	144.0	2.465	.836	2.444
8	4.1 No Yawsteering C=1 Copps MSA	144.4	51.0	143.8	2.464	.835	2.443
9	4.1 Linear Yaw-steering C=1 Copps MSA	144.5	50.9	143.8	2.464	.835	2.443

TABLE 8 (Cont'd)

Standard Deviations of the Uncertainty of the Errors at the End of TEI

Case No.	Case Description	U (feet)	V (feet)	W (feet)	$\dot{U}$ (fps)	$\dot{V}$ (fps)	$\dot{W}$ (fps)
10	4.1 Quadratic Yawsteering C=1 Copps MSA	145.1	51.1	144.1	2.465	.836	2.444
11	4.1 No Yawsteering C=.5 Copps MSA	144.7	50.8	143.8	2.465	.836	2.444
12	4.1 Linear Yaw-steering C=.5 Copps MSA	144.3	50.6	143.9	2.465	.836	2.444
13	4.4 No Yawsteering C=1 Copps Entry	146.8	53.4	146.3	2.288	.796	2.277
14	4.4 Linear Yaw-steering C=1 Copps Entry	146.1	53.4	146.3	2.288	.796	2.277
15	4.5 No Yawsteering C=1 Copps Entry	125.1	41.9	125.2	2.234	.717	2.228
16	4.5 Linear Yaw-steering C=1 Copps Entry	125.1	42.0	125.2	2.234	.717	2.228

TABLE 9

First Midcourse Correction ΔV Penalties (f.p.s.)
(Due to Inertial Platform and Performance Errors Only)

Occurs at MSA

CASE NO.	CASE DESCRIPTION	Mag. of Midcourse 99.73%	INMISY 1σ	INMISZ 1σ	ASFX 1σ	GCDY 1σ	GCDZ 1σ	ACBIAX 1σ	ACBIAY 1σ	ACBIAZ 1σ	TIMCOU 1σ
1	4.1 C=1 No Yaw-steering Copps Entry	9.193	.567	1.045	.879	.943	1.733	1.848	.546	.293	1.255
2	4.1 C=1 Linear Yaw-steering Copps Entry	9.183	.567	1.045	.882	.943	1.732	1.841	.544	.292	1.254
3	4.1 C=1 Quadratic Copps Entry	9.210	.567	1.026	.886	.944	1.701	1.850	.545	.227	1.268
4	4.1 C=.5 No Yaw-steering Copps Entry	9.245	.568	1.046	.887	.944	1.732	1.867	.571	.308	1.261
5	4.1 C=.5 Linear Yaw-steering Copps Entry	9.151	.567	1.046	.881	.944	1.733	1.799	.572	.342	1.241
6	4.1 C=1. Linear Yaw-steering VG/VG Entry	9.155	.566	1.046	.887	.942	1.732	1.818	.544	.342	1.241
7	4.1 C=.5 Linear Yaw-steering VG/VG Entry	9.238	.568	1.046	.926	.943	1.732	1.848	.573	.337	1.246
8	4.1 C=1.0 No Yaw-steering Copps MSA	9.200	.567	1.046	.888	.943	1.733	1.847	.546	.294	1.255

TABLE 9 (Cont'd)

First Midcourse Correction ΔV Penalties (f.p.s.)
(Due to Inertial Platform and Performance Errors Only)

Occurs at MSA

CASE NO.	CASE DESCRIPTION	Mag. of Midcourse 99.73%	INMISY 1σ	INMISZ 1σ	ASFX 1σ	GCDY 1σ	GCDZ 1σ	ACBIAX 1σ	ACBIAY 1σ	ACBIAZ 1σ	TIMCOU 1σ
9	4.1 C=1.0 Linear Yaw-steering Copps MSA	9.193	.567	1.046	.885	.943	1.733	1.845	.547	.292	1.254
10	4.1 C=1. Quadratic Yawsteering Copps MSA	9.212	.568	1.026	.870	.944	1.701	1.858	.550	.232	1.268
11	4.1 C=.5 No Yaw-steering Copps MSA	9.201	.568	1.046	.884	.944	1.732	1.832	.572	.311	1.262
12	4.1 C=.5 Linear Yaw-steering Copps MSA	9.203	.567	1.046	.882	.943	1.732	1.839	.573	.293	1.260
13	4.4 C=1.0 No Yaw-steering Copps Entry	9.704	.217	.936	.961	.362	1.559	2.377	.574	.146	1.177
14	4.4 C=1.0 Linear Yaw-steering Copps Entry	9.645	.217	.935	.932	.362	1.557	2.361	.581	.150	1.176
15	4.5 C=1.0 Linear Yaw-steering Copps Entry	8.958	.276	.865	.924	.457	1.429	2.033	.460	.151	1.387
16	4.5 C=1.0 Linear Yaw-steering Copps Entry	8.979	.256	.865	.926	.457	1.428	2.044	.458	.150	1.388

TABLE 10

Second Midcourse Correction Penalties (f.p.s.)
(Due to Inertial Platform and Performance Errors Only)

Occurs at Entry (400,000 feet)

CASE NO.	CASE DESCRIPTION	Mag. of Midcourse 99.73%	INMISY 1σ	INMISZ 1σ	ASFX 1σ	AMSZXZ 1σ	GCDY 1σ	GCDZ 1σ	ACBIAX 1σ	ACBIAY 1σ	ACBIAZ 1σ	TIMCOU 1σ
1	4.1 C=1 No Yaw-steering Copps Entry	13.026	1.766	.720	.781	.412	2.933	1.193	1.640	.339	.942	1.089
2	4.1 C=1 Linear Yaw-steering Copps Entry	13.019	1.765	.720	.782	.415	2.932	1.192	1.634	.342	.945	1.086
3	4.1 C=1 Quadratic Copps Entry	13.094	1.766	.713	.842	.291	2.934	1.181	1.767	.344	.673	1.160
4	4.1 C=.5 No Yaw-steering Copps Entry	13.062	1.767	.720	.780	.424	2.935	1.189	1.637	.393	.958	1.102
5	4.1 C=.5 Linear Yaw-steering Copps Entry	13.004	1.766	.721	.779	.415	2.935	1.192	1.598	.394	.948	1.094
6	4.1 C=1. Linear Yaw-steering $\dot{V}G/VG$ Entry	12.977	1.765	.719	.774	.435	2.932	1.190	1.591	.344	.998	1.043

TABLE 10 (Cont'd)

Second Midcourse Correction Penalties (f.p.s.)
(Due to Inertial Platform and Performance Errors Only)

Occurs at Entry (400,000 feet)

CASE NO.	CASE DESCRIPTION	Mag. of Midcourse 99.73%	INMISY 1σ	INMISZ 1σ	ASFX 1σ	AMSZXZ 1σ	GCDY 1σ	GCDZ 1σ	ACBIAX 1σ	ACBIAY 1σ	ACBIAZ 1σ	TIMCOU 1σ
7	4.1 C=.5 Linear Yaw-steering VG/VG Entry	13.022	1.768	.720	.797	.435	2.935	1.189	1.607	.389	.993	1.050
8	4.1 C=1.0 No Yaw-steering Copps MSA	13.029	1.766	.720	.785	.413	2.933	1.193	1.636	.345	.947	1.089
9	4.1 C=1.0 Linear Yaw-steering Copps MSA	13.024	1.766	.722	.785	.413	2.932	1.194	1.636	.342	.945	1.086
10	4.1 C=1. No Yaw-steering Copps MSA	13.090	1.767	.713	.830	.282	2.934	1.180	1.772	.339	.667	1.160
11	4.1 C=.5 No Yaw-steering Copps MSA	13.039	1.768	.720	.777	.424	2.935	1.190	1.612	.397	.965	1.102
12	4.1 C=.5 Linear Yaw-steering Copps MSA	13.032	1.766	.721	.780	.414	2.935	1.191	1.626	.396	.946	1.094

TABLE 10 (Cont'd)

CASE NO.	CASE DESCRIPTION	Second Midcourse Correction Penalties (f.p.s.) (Due to Inertial Platform and Performance Errors Only) Occurs at Entry (400,000 feet)										
		Mag. of Midcourse 99.73%	INMISY 1σ	INMISZ 1σ	ASFX 1σ	AMSZXZ 1σ	GCDY 1σ	GCDZ 1σ	ACBIAX 1σ	ACBIAY 1σ	ACBIAZ 1σ	TIMCOU 1σ
13	4.4 No Yaw-steering C=1.0 Copps Entry	8.596	1.338	.271	.362	.316	2.235	.451	.897	.152	.887	.443
14	4.4 Linear Yaw-steering C-1.0 Copps Entry	8.595	1.338	.270	.352	.319	2.235	.450	.891	.146	.890	.443
15	4.5 No Yaw-steering C=1.0 Copps Entry	9.065	1.432	.195	.390	.337	2.369	.322	.861	.092	.804	.584
16	4.5 Linear Yaw-steering C=1.0 Copps Entry	9.073	1.432	.195	.391	.342	2.369	.321	.865	.092	.811	.585

TABLE 11

Midcourse Correction Penalties due to
Inertial Platform and Performance Errors

Includes Effects of Tripled Sextant Errors
in Lunar Orbit Navigation

Case No.	Case Description (All Trajectory 4.1)	First Midcourse 99.73% Point (fps)	Second Midcourse 99.73% Point (fps)
1	Entry, No Yaw C=1.0	9.227	13.178
2	Entry Lin Yaw C=1.0	9.217	13.172
4	Entry No Yaw C=0.5	9.280	13.214
5	Entry Lin Yaw C=0.5	9.185	13.156
8	MSA No Yaw C=1.0	9.234	13.411
9	MSA Lin Yaw C=1.0	9.227	13.175
11	MSA No Yaw C=0.5	9.236	13.195
12	MSA Lin Yaw C=0.5	9.237	13.184